Advanced Percolation Solution for Hopping Conductivity

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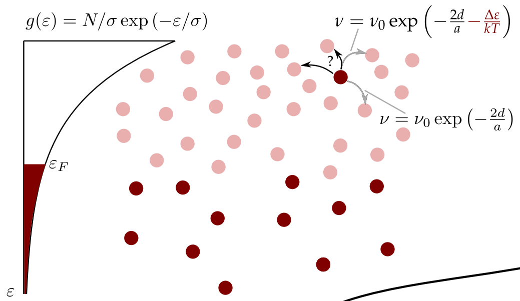
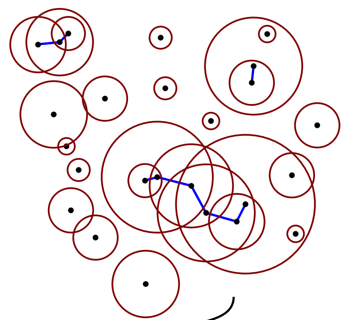
4D variable range hopping problem



3D percolation problem



4D VRH ⟷ 3D percolation Mapping

Map Miller-Abrahams hopping rates onto resistor network

$$R_{ij} = \frac{kT}{e^2\nu_0} \exp\left(\frac{2d_{ij}}{a} + \frac{\max(\varepsilon_i,\varepsilon_j)-\varepsilon_F}{kT}\right)$$

Formulate percolation solution of the resistor network

$$R_{\text{crit}} = \frac{kT}{e^2\nu_0} \exp\left(\frac{\varepsilon^*-\varepsilon_F}{kT}\right)$$

... with critical *energy* ε^* and the bonding criteria

$$\frac{2d_{ij}}{a} + \frac{\varepsilon_j}{kT} \le \frac{\varepsilon^*}{kT} \qquad \frac{2d_{ij}}{a} + \frac{\varepsilon_i}{kT} \le \frac{\varepsilon^*}{kT}$$

Define site *radius* $r_i = \frac{a}{2kT}(\varepsilon^* - \varepsilon_i)$. Bonding criteria:

$$d_{ij} \le r_j \qquad d_{ij} \le r_i$$

Distribution function of site *radii*

$$|g(r)dr| = |g(\varepsilon)d\varepsilon| \quad \Rightarrow \quad g(r) = \frac{N}{\sigma}\exp(\varepsilon/\sigma)\,\frac{2kT}{a}$$

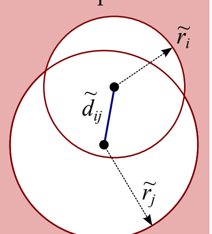
Introduce dimensionless quantities $\tilde{r}_i, \tilde{d}_{ij}, \tilde{g}(\tilde{r})$

$$\tilde{g}(\tilde{r}) = n\exp(-\tilde{r}) \qquad \tilde{d}_{ij} \le \tilde{r}_j \qquad \tilde{d}_{ij} \le \tilde{r}_i$$

New universal, purely geometrical percolation problem of spheres with distributed radii:

Find critical concentration n_c of spheres for the infinite percolating cluster (result see right column).

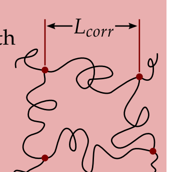
$$n = n_c \approx 0.219$$



Solve for ε^* and R_{crit}

$$R_{\text{crit}} = \frac{kT}{e^2\nu_0}\left[n_c\left(\frac{2kT}{a\sigma}\right)^3 n_e^{-1}\right]^{\sigma/kT}$$

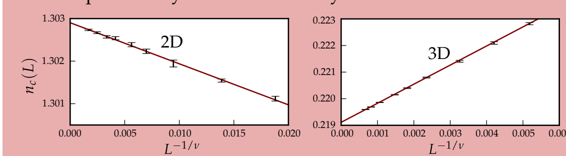
Find resistivity with the correlation length of the critical subnetwork: $\rho = L_{corr}R_{crit}$



$$\rho = A\frac{a\sigma}{e^2\nu_0}\left(\frac{\sigma}{kT}\right)^\nu \left[8n_c\left(\frac{kT}{a\sigma}\right)^3 n_e^{-1}\right]^{\sigma/kT}$$

Solution 3D percolation problem

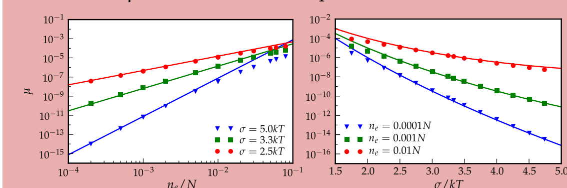
Numerical solution of the new percolating spheres problem with exponentially distributed radii yields:



$$n_c^{2D} = 1.30(3) \qquad n_c^{3D} = 0.21(9)$$

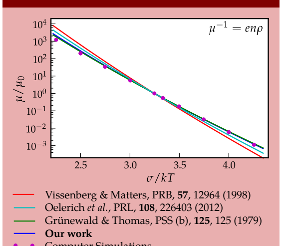
Numerical prefactor

Fit result of ρ to Monte-Carlo computer simulation results:

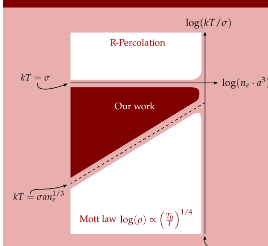


$$A = 0.3(6)$$

Literature comparison



Range of validity



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