

In[1350]:=

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Off[General::spell1]
Print["Lösungen Übungsblatt 8, zum 27.06.2006"]

Print["1."]
A =  $\begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 3 \end{pmatrix}$ ;
Asym =  $\frac{1}{2} * (A + \text{Transpose}[A])$ ;
Ass =  $\frac{1}{2} * (A - \text{Transpose}[A])$ ;
Print[" A =  $\frac{1}{2} (A+A^T) + \frac{1}{2} (A-A^T) =$ ", Asym // MatrixForm,
" + ", Ass // MatrixForm, " = ", Asym + Ass // MatrixForm]
Clear[A, Asym, Ass]

Print["2."]

O1 =  $\frac{1}{\sqrt{2}} * \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ ;
O2 =  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ;
O3 =  $\frac{1}{2} * \begin{pmatrix} \sqrt{3} & 1 \\ 1 & -\sqrt{3} \end{pmatrix}$ ;

Print[" a)"]
O11 = O1.O1;
O12 = O1.O2;
O13 = O1.O3;
O22 = O2.O2;
O21 = O2.O1;
O23 = O2.O3;
O33 = O3.O3;
O31 = O3.O1;
O32 = O3.O2;
Print[" O11 * O11T = ", O11 // MatrixForm, ".",
Transpose[O11] // MatrixForm, " = ", Simplify[O11.Transpose[O11]] // MatrixForm]
Print[" O12 * O12T = ", O12 // MatrixForm, ".", Transpose[O12] // MatrixForm,
" = ", Simplify[O12.Transpose[O12]] // MatrixForm]
Print[" O13 * O13T = ", O13 // MatrixForm, ".", Transpose[O13] // MatrixForm,
" = ", Simplify[O13.Transpose[O13]] // MatrixForm]
Print[" O22 * O22T = ", O22 // MatrixForm, ".", Transpose[O22] // MatrixForm,
" = ", Simplify[O22.Transpose[O22]] // MatrixForm]
Print[" O21 * O21T = ", O21 // MatrixForm, ".", Transpose[O21] // MatrixForm,
" = ", Simplify[O21.Transpose[O21]] // MatrixForm]
Print[" O23 * O23T = ", O23 // MatrixForm, ".", Transpose[O23] // MatrixForm,
" = ", Simplify[O23.Transpose[O23]] // MatrixForm]
Print[" O33 * O33T = ", O33 // MatrixForm, ".", Transpose[O33] // MatrixForm,
" = ", Simplify[O33.Transpose[O33]] // MatrixForm]
Print[" O31 * O31T = ", O31 // MatrixForm, ".", Transpose[O31] // MatrixForm,
" = ", Simplify[O31.Transpose[O31]] // MatrixForm]
Print[" O32 * O32T = ", O32 // MatrixForm, ".", Transpose[O32] // MatrixForm,
" = ", Simplify[O32.Transpose[O32]] // MatrixForm]

Print[" b)"]
Print[" det O1 = ", Det[O1], " -> ist Element von SO(2)"]
Print[" det O2 = ", Det[O2], " -> ist Element von SO(2)"]
Print[" det O3 = ", Det[O3]]

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Print[" c)"]
r = {1, 0};
Print[" O1.ŕ = ", O1 // MatrixForm, ".", r // MatrixForm, " = ", O1.r // MatrixForm]
ListPlot[{{0, 0}, r, O1.r}, PlotStyle -> PointSize[0.02],
  PlotRange -> {{-1.1, 1.1}, {-1.1, 1.1}}, AspectRatio -> 1];
Print[" acos(ŕ.(O1.ŕ)) = acos(", r.(r.O1), ") = ",
  ArcCos[r.(r.O1)], " Obacht! (cos( $\frac{\pi}{4}$ ) == cos( $\frac{7\pi}{4}$ ))"]

Print[" O2.ŕ = ", O2 // MatrixForm, ".", r // MatrixForm, " = ", O2.r // MatrixForm]
ListPlot[{{0, 0}, r, O2.r}, PlotStyle -> PointSize[0.02],
  PlotRange -> {{-1.1, 1.1}, {-1.1, 1.1}}, AspectRatio -> 1];
Print[" acos(ŕ.(O2.ŕ)) = acos(", r.(r.O2), ") = ", ArcCos[r.(r.O2)]]

Print[" O3.ŕ = ", O3 // MatrixForm, ".", r // MatrixForm, " = ", O3.r // MatrixForm]
ListPlot[{{0, 0}, r, O3.r}, PlotStyle -> PointSize[0.02],
  PlotRange -> {{-1.1, 1.1}, {-1.1, 1.1}}, AspectRatio -> 1];
Print[" acos(ŕ.(O3.ŕ)) = acos(", r.(r.O3), ") = ", ArcCos[r.(r.O3)]]

Clear[O1, O2, O3, O11, O12, O13, O21, O22, O23, O31, O32, O33, r]

Print["3."]
U1 =  $\frac{1}{\sqrt{2}}$  *  $\begin{pmatrix} 1 & 1 \\ -I & I \end{pmatrix}$ ;
U2 =  $\begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$ ;
U3 =  $\frac{1}{\sqrt{2}}$  *  $\begin{pmatrix} I & I \\ I & -I \end{pmatrix}$ ;

Print[" a)"]
U11 = U1.U1;
U12 = U1.U2;
U13 = U1.U3;
U22 = U2.U2;
U21 = U2.U1;
U23 = U2.U3;
U33 = U3.U3;
U31 = U3.U1;
U32 = U3.U2;
Print[" U11 * U11† = ", U11 // MatrixForm,
  ".", Conjugate[Transpose[U11]] // MatrixForm, " = ",
  Simplify[U11.Conjugate[Transpose[U11]]] // MatrixForm]
Print[" U12 * U12† = ", U12 // MatrixForm, ".", Conjugate[Transpose[U12]] // MatrixForm,
  " = ", Simplify[U12.Conjugate[Transpose[U12]]] // MatrixForm]
Print[" U13 * U13† = ", U13 // MatrixForm, ".", Conjugate[Transpose[U13]] // MatrixForm,
  " = ", Simplify[U13.Conjugate[Transpose[U13]]] // MatrixForm]
Print[" U22 * U22† = ", U22 // MatrixForm, ".", Conjugate[Transpose[U22]] // MatrixForm,
  " = ", Simplify[U22.Conjugate[Transpose[U22]]] // MatrixForm]
Print[" U21 * U21† = ", U21 // MatrixForm, ".", Conjugate[Transpose[U21]] // MatrixForm,
  " = ", Simplify[U21.Conjugate[Transpose[U21]]] // MatrixForm]
Print[" U23 * U23† = ", U23 // MatrixForm, ".", Conjugate[Transpose[U23]] // MatrixForm,
  " = ", Simplify[U23.Conjugate[Transpose[U23]]] // MatrixForm]
Print[" U33 * U33† = ", U33 // MatrixForm, ".", Conjugate[Transpose[U33]] // MatrixForm,
  " = ", Simplify[U33.Conjugate[Transpose[U33]]] // MatrixForm]
Print[" U31 * U31† = ", U31 // MatrixForm, ".", Conjugate[Transpose[U31]] // MatrixForm,
  " = ", Simplify[U31.Conjugate[Transpose[U31]]] // MatrixForm]
Print[" U32 * U32† = ", U32 // MatrixForm, ".", Conjugate[Transpose[U32]] // MatrixForm,
  " = ", Simplify[U32.Conjugate[Transpose[U32]]] // MatrixForm]

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Print[" b)"]
Print["  det U1 = ", Det[U1]]
Print["  det U2 = ", Det[U2]]
Print["  det U3 = ", Det[U3], " -> ist Element von SU(2)"]

Clear[U1, U2, U3, U11, U12, U13, U21, U22, U23, U31, U32, U33]

Print["4."]

$$M = \begin{pmatrix} e & 2 & 1 & 3 \\ 7 & 0 & 5 & 4 \\ \frac{1}{8} & 0 & 13 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

Print["  det", M // MatrixForm, " = ", Det[M]]
Clear[M]

Print["5."]

$$A = \frac{1}{2} * \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix};$$


$$B = \frac{1}{4} * \begin{pmatrix} 3 & -I \\ I & 3 \end{pmatrix};$$


Print[" a)"]
Print["  Eigenwerte von A: ", Eigenvalues[A]]
Print["  Eigenwerte von B: ", Eigenvalues[B]]

Print[" b)"]
Print["  tr A = ", Tr[A], " = Summe Eigenwerte"]
Print["  det A = ", Det[A], " = Produkt Eigenwerte"]
Print["  tr B = ", Tr[B], " = Summe Eigenwerte"]
Print["  det B = ", Det[B], " = Produkt Eigenwerte"]

Print[" c)"]
Print["  Eigenvektoren von A: ", Eigenvectors[A]]

$$X1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix};$$

Print["  X = ", X1 // MatrixForm]
Print["  X.XT = ", X1.Transpose[X1] // MatrixForm,
" = ", Transpose[X1].X1 // MatrixForm, " = XT.X"]

Print["  Eigenvektoren von B: ", Eigenvectors[B]]

$$X2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -I & I \\ 1 & 1 \end{pmatrix};$$

Print["  X = ", X2 // MatrixForm]
Print["  X.X* = ", X2.Conjugate[Transpose[X2]] // MatrixForm,
" = ", Conjugate[Transpose[X2]].X2 // MatrixForm, " = X*.X"]

Print[" d)"]

$$e\Lambda1 = \begin{pmatrix} e^2 & 0 \\ 0 & e \end{pmatrix};$$

Print["  exp A = O.exp  $\Lambda_A$ .OT = ", X1 // MatrixForm, ".", e\Lambda1 // MatrixForm,
".", Transpose[X1] // MatrixForm, " = ", X1.e\Lambda1.Transpose[X1] // MatrixForm]

$$e\Lambda2 = \begin{pmatrix} e & 0 \\ 0 & e^{\frac{1}{2}} \end{pmatrix};$$

Print["  exp B = U.exp  $\Lambda_B$ .U* = ", X2 // MatrixForm,
".", e\Lambda2 // MatrixForm, ".", Conjugate[Transpose[X2]] // MatrixForm,
" = ", X2.e\Lambda2.Conjugate[Transpose[X2]] // MatrixForm]

Clear[A, B, X1, X2, e\Lambda1, e\Lambda2]

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Lösungen Übungsblatt 8, zum 27.06.2006

1.

$$A = \frac{1}{2}(A+A^T) + \frac{1}{2}(A-A^T) = \begin{pmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 0 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 3 \end{pmatrix} + \begin{pmatrix} 0 & \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 2 \\ 4 & 1 & 3 \end{pmatrix}$$

2.

a)

$$O11 * O11^T = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O12 * O12^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O13 * O13^T = \begin{pmatrix} \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} \\ -\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} \\ -\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O22 * O22^T = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O21 * O21^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O23 * O23^T = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O33 * O33^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O31 * O31^T = \begin{pmatrix} \frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{3}}{2} - \frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} \\ \frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} & -\frac{\sqrt{3}}{2} + \frac{1}{2\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$O32 * O32^T = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

b)

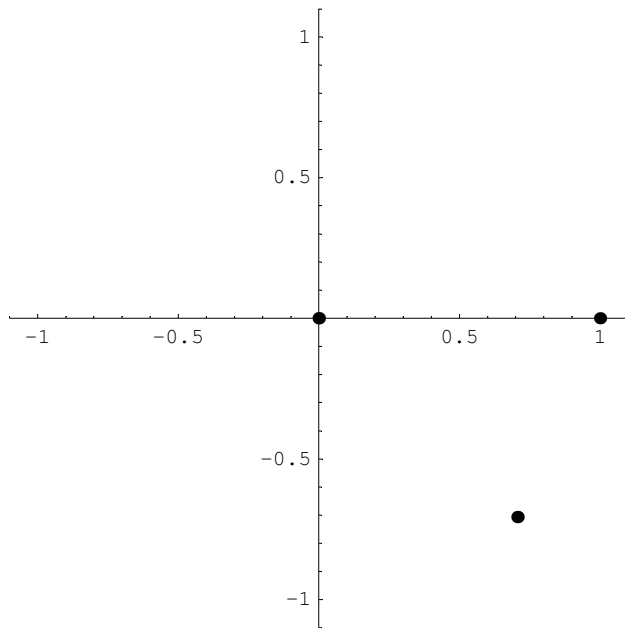
det O1 = 1 -> ist Element von SO(2)

det O2 = 1 -> ist Element von SO(2)

det O3 = -1

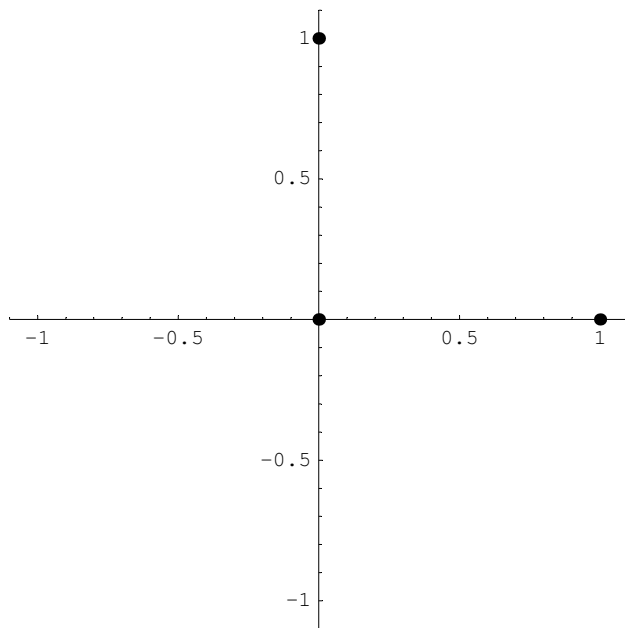
c)

$$O1.\hat{f} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$



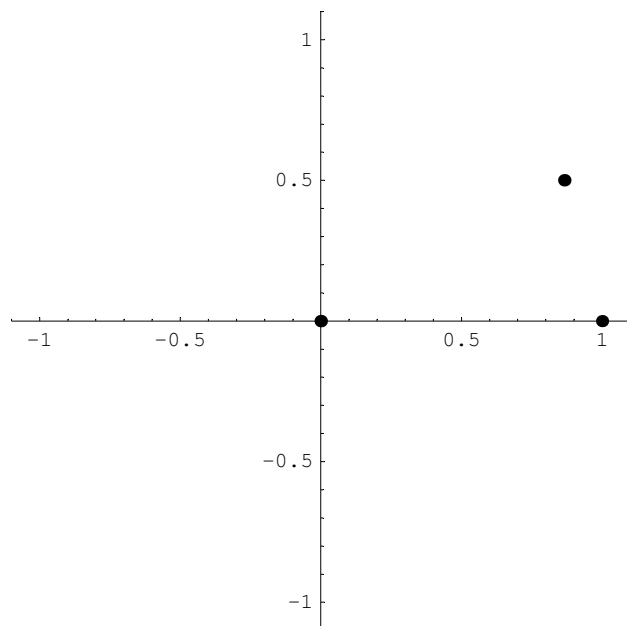
$$\text{acos}(\hat{f} \cdot (O1.\hat{f})) = \text{acos}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \text{ Obacht! } (\cos\left(\frac{\pi}{4}\right) = \cos\left(\frac{7\pi}{4}\right))$$

$$O2.\hat{f} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



$$\text{acos}(\hat{f} \cdot (O2.\hat{f})) = \text{acos}(0) = \frac{\pi}{2}$$

$$03. \hat{f} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$$



$$\text{acos}(\hat{f} \cdot (03. \hat{f})) = \text{acos}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

3.

a)

$$U11 * U11^+ = \begin{pmatrix} \frac{1}{2} - \frac{i}{2} & \frac{1}{2} + \frac{i}{2} \\ \frac{1}{2} - \frac{i}{2} & -\frac{1}{2} - \frac{i}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} + \frac{i}{2} & \frac{1}{2} + \frac{i}{2} \\ \frac{1}{2} - \frac{i}{2} & -\frac{1}{2} + \frac{i}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U12 * U12^+ = \begin{pmatrix} \frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U13 * U13^+ = \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -i & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U22 * U22^+ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U21 * U21^+ = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U23 * U23^+ = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U33 * U33^+ = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U31 * U31^+ = \begin{pmatrix} \frac{1}{2} + \frac{i}{2} & -\frac{1}{2} + \frac{i}{2} \\ -\frac{1}{2} + \frac{i}{2} & \frac{1}{2} + \frac{i}{2} \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{2} - \frac{i}{2} & -\frac{1}{2} - \frac{i}{2} \\ -\frac{1}{2} - \frac{i}{2} & \frac{1}{2} - \frac{i}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$U32 * U32^+ = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

b)

$$\det U1 = i$$

$$\det U2 = -1$$

$$\det U3 = 1 \rightarrow \text{ist Element von } SU(2)$$

4.

$$\det \begin{pmatrix} e & 2 & 1 & 3 \\ 7 & 0 & 5 & 4 \\ \frac{1}{8} & 0 & 13 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} = 1$$

5.

a)

Eigenwerte von A: $\{2, 1\}$ Eigenwerte von B: $\{1, \frac{1}{2}\}$

b)

 $\text{tr A} = 3 = \text{Summe Eigenwerte}$ $\det A = 2 = \text{Produkt Eigenwerte}$ $\text{tr B} = \frac{3}{2} = \text{Summe Eigenwerte}$ $\det B = \frac{1}{2} = \text{Produkt Eigenwerte}$

c)

Eigenvektoren von A: $\{\{1, 1\}, \{-1, 1\}\}$

$$X = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$X \cdot X^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = X^T \cdot X$$

Eigenvektoren von B: $\{\{-i, 1\}, \{i, 1\}\}$

$$X = \begin{pmatrix} -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$X \cdot X^+ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = X^+ \cdot X$$

d)

$$\exp A = O \cdot \exp \Lambda_A \cdot O^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} e^2 & 0 \\ 0 & e \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{e}{2} + \frac{e^2}{2} & -\frac{e}{2} + \frac{e^2}{2} \\ -\frac{e}{2} + \frac{e^2}{2} & \frac{e}{2} + \frac{e^2}{2} \end{pmatrix}$$

$$\exp B = U \cdot \exp \Lambda_B \cdot U^+ = \begin{pmatrix} -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \cdot \begin{pmatrix} e & 0 \\ 0 & \sqrt{e} \end{pmatrix} \cdot \begin{pmatrix} \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{e}}{2} + \frac{e}{2} & \frac{i\sqrt{e}}{2} - \frac{i}{2} \\ -\frac{i\sqrt{e}}{2} + \frac{i}{2} & \frac{\sqrt{e}}{2} + \frac{e}{2} \end{pmatrix}$$