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In[1]:= Off[General::spelll]

In[3347]:=
        Print["Lösungen Übungsblatt 6, zum 06.06.2006"]

        Print["1."]
        Print[" a)"]
        f[x_, y_] := x^2 + 2 * x * y + 3 * y^2;
        gradf = Simplify[{∂x f[x, y], ∂y f[x, y]}];
        H = Simplify[( ∂x,x f[x, y]  ∂x,y f[x, y]
                       ∂y,x f[x, y]  ∂y,y f[x, y] )];
        Hxy[x_, y_] = H;

        Print["   ∇f(x,y)  = ", gradf // MatrixForm];
        Print["   ∇f(x,y) == 0 => ", Solve[{gradf[[1]] == 0, gradf[[2]] == 0}, {x, y}]]
        Print["   H  = ", H // MatrixForm];
        Print["   H(0,0)  = ", Hxy[0, 0] // MatrixForm];
        Print["   detH(0,0)  = ", Det[Hxy[0, 0]], " > 0 => definit."]
        Print["   Mit fxx > 0 => positiv definit => Minimum in (0,0)"]
        Clear[f, gradf, H, Hxy];

        Print[" b)"]
        f[x_, y_] := 1 / Sqrt[x^2 + 2 * y^2 + 1];
        gradf = Simplify[{∂x f[x, y], ∂y f[x, y]}];
        H = Simplify[( ∂x,x f[x, y]  ∂x,y f[x, y]
                       ∂y,x f[x, y]  ∂y,y f[x, y] )];
        Hxy[x_, y_] = H;

        Print["   ∇f(x,y)  = ", gradf // MatrixForm];
        Print["   ∇f(x,y) == 0 => ", Solve[{gradf[[1]] == 0, gradf[[2]] == 0}, {x, y}]]
        Print["   H  = ", H // MatrixForm];
        Print["   H(0,0)  = ", Hxy[0, 0] // MatrixForm];
        Print["   Alle Eigenwerte < 0 => negativ definit => Maximum in (0,0)"]
        Clear[f, gradf, H, Hxy];

        Print[" c)"]
        f[x_, y_] := x^2 - x * y + y^3;
        gradf = Simplify[{∂x f[x, y], ∂y f[x, y]}];
        H = Simplify[( ∂x,x f[x, y]  ∂x,y f[x, y]
                       ∂y,x f[x, y]  ∂y,y f[x, y] )];
        Hxy[x_, y_] = H;

        Print["   ∇f(x,y)  = ", gradf // MatrixForm];
        Print["   ∇f(x,y) == 0 => ", Solve[{gradf[[1]] == 0, gradf[[2]] == 0}, {x, y}]]
        Print["   H  = ", H // MatrixForm];
        Print["   H(0,0)  = ", Hxy[0, 0] // MatrixForm];
        Print["   detH(0,0)  = ", Det[Hxy[0, 0]], " < 0 => indefinit => Sattelpunkt in (0,0)."]
        Print["   H(1/12, 1/6)  = ", Hxy[1/12, 1/6] // MatrixForm];
        Print["   detH(1/12, 1/6)  = ", Det[Hxy[1/12, 1/6]], " > 0 => definit."]
        Print["   Mit fxx > 0 => positiv definit => Minimum in (1/12, 1/6)"]
        Clear[f, gradf, H, Hxy];

        Print["2."]
        f[x_, y_] := x^2 + 2 * x * y + 3 * y^2;
        dfx[x_, y_] = ∂x f[x, y];
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dfy[x_, y_] = ∂y f[x, y];
dfxx[x_, y_] = ∂x,x f[x, y];
dfxy[x_, y_] = ∂x,y f[x, y];
dfyy[x_, y_] = ∂y,y f[x, y];
taylor = f[x0, y0] + (x - x0) * dfx[x0, y0] + (y - y0) * dfy[x0, y0] +
   (x - x0)^2/2 * dfxx[x0, y0] + (x - x0) * (y - y0) * dfxy[x0, y0] + (y - y0)^2/2 * dfyy[x0, y0];
Print["   f(x,y)x0,y0  = ", taylor]
Print["                 = ", Simplify[taylor]]
Clear[f, dfx, dfy, dfxx, dfxy, dfyy, taylor];

Print["3."]
f[x_, y_] := 1/Sqrt[x^2 + 2 * y^2 + 1];
dfx[x_, y_] = ∂x f[x, y];
dfy[x_, y_] = ∂y f[x, y];
dfxx[x_, y_] = ∂x,x f[x, y];
dfxy[x_, y_] = ∂x,y f[x, y];
dfyy[x_, y_] = ∂y,y f[x, y];
taylor[x0_, y0_] := f[x0, y0] + (x - x0) * dfx[x0, y0] + (y - y0) * dfy[x0, y0] +
   (x - x0)^2/2 * dfxx[x0, y0] + (x - x0) * (y - y0) * dfxy[x0, y0] + (y - y0)^2/2 * dfyy[x0, y0];
Print[" a)"]
Print["   f(x,y)1,-1  = f̃(x,y)  = ", taylor[1, -1]]
Print["                          = ", Simplify[taylor[1, -1]]]
Print["   Test:"]
taylorxya[x_, y_] = Simplify[taylor[1, -1]];
Print["   f(1,-1)  = ", f[1, -1]]
Print["   f̃(1,-1)  = ", taylorxya[1, -1]]

Print[" b)"]
Print["   f(x,y)0,0  = ", taylor[0, 0]]
Print["   Test:"]
taylorxyb[x_, y_] = Simplify[taylor[0, 0]];
Print["   f(0,0)  = ", f[0, 0]]
Print["   f̃(0,0)  = ", taylorxyb[0, 0]]

p1 = Plot3D[f[x, y], {x, -2, 2}, {y, -2, 2}, AxesLabel → {"x-Achse", "y-Achse", "z-Achse"}];
p2 = Plot3D[taylorxya[x, y], {x, -0.5, 1.5},
   {y, -2, 0.5}, AxesLabel → {"x-Achse", "y-Achse", "z-Achse"}];
p3 = Plot3D[taylorxyb[x, y], {x, -0.7, 0.7}, {y, -0.7, 0.7},
   AxesLabel → {"x-Achse", "y-Achse", "z-Achse"}];
p4 = Show[p2, p1, ViewPoint → {2, 1.5, 1.5}, AxesLabel → {"x-Achse", "y-Achse", "z-Achse"}];
p5 = Show[p3, p1, ViewPoint → {2, 1.5, -1.2}, AxesLabel → {"x-Achse", "y-Achse", "z-Achse"}];
Clear[f, dfx, dfy, dfxx, dfxy, dfyy, taylor];
```

Lösungen Übungsblatt 6, zum 06.06.2006

1.

a)

$\nabla f(x,y) = \begin{pmatrix} 2\,(x+y) \\ 2\,(x+3\,y) \end{pmatrix}$

$\nabla f(x,y) == 0 \Rightarrow \{\{x\to 0,\ y\to 0\}\}$

$\mathbb{H} = \begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix}$

$\mathbb{H}(0,0) = \begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix}$

$\det\mathbb{H}(0,0) = 8 > 0 \Rightarrow$ definit.

Mit $f_{xx} > 0 \Rightarrow$ positiv definit => Minimum in (0,0)

b)

$\nabla f(x,y) = \begin{pmatrix} -\frac{x}{(1+x^2+2\,y^2)^{3/2}} \\ -\frac{2\,y}{(1+x^2+2\,y^2)^{3/2}} \end{pmatrix}$

$\nabla f(x,y) == 0 \Rightarrow \{\{x\to 0,\ y\to 0\}\}$

$\mathbb{H} = \begin{pmatrix} \frac{-1+2\,x^2-2\,y^2}{(1+x^2+2\,y^2)^{5/2}} & \frac{6\,x\,y}{(1+x^2+2\,y^2)^{5/2}} \\ \frac{6\,x\,y}{(1+x^2+2\,y^2)^{5/2}} & -\frac{2\,(1+x^2-4\,y^2)}{(1+x^2+2\,y^2)^{5/2}} \end{pmatrix}$

$\mathbb{H}(0,0) = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

Alle Eigenwerte < 0 => negativ definit => Maximum in (0,0)

c)

$\nabla f(x,y) = \begin{pmatrix} 2\,x-y \\ -x+3\,y^2 \end{pmatrix}$

$\nabla f(x,y) == 0 \Rightarrow \left\{\{x\to 0,\ y\to 0\},\ \left\{x\to \frac{1}{12},\ y\to \frac{1}{6}\right\}\right\}$

$\mathbb{H} = \begin{pmatrix} 2 & -1 \\ -1 & 6\,y \end{pmatrix}$

$\mathbb{H}(0,0) = \begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$

$\det\mathbb{H}(0,0) = -1 < 0 \Rightarrow$ indefinit => Sattelpunkt in (0,0).

$\mathbb{H}(\frac{1}{12},\frac{1}{6}) = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

$\det\mathbb{H}(\frac{1}{12},\frac{1}{6}) = 1 > 0 \Rightarrow$ definit.

Mit $f_{xx} > 0 \Rightarrow$ positiv definit => Minimum in $(\frac{1}{12},\frac{1}{6})$

2.

$$f(x,y)_{x0,y0} = (x-x0)^2 + x0^2 + 2\,(x-x0)\,(y-y0) + 3\,(y-y0)^2 + 2\,x0\,y0 + 3\,y0^2 + (x-x0)\,(2\,x0+2\,y0) + (y-y0)\,(2\,x0+6\,y0)$$

$$= x^2 + 2\,x\,y + 3\,y^2$$

3.

a)

$$f(x,y)_{1,-1} = \tilde{f}(x,y) = \frac{1}{2} + \frac{1-x}{8} - \frac{1}{64}(-1+x)^2 + \frac{1+y}{4} - \frac{3}{16}(-1+x)(1+y) + \frac{1}{16}(1+y)^2$$

$$= \frac{1}{64}(71 - x^2 + 36\,y + 4\,y^2 - 6\,x\,(3+2\,y))$$

Test:

$f(1,-1) = \frac{1}{2}$

$\tilde{f}(1,-1) = \frac{1}{2}$

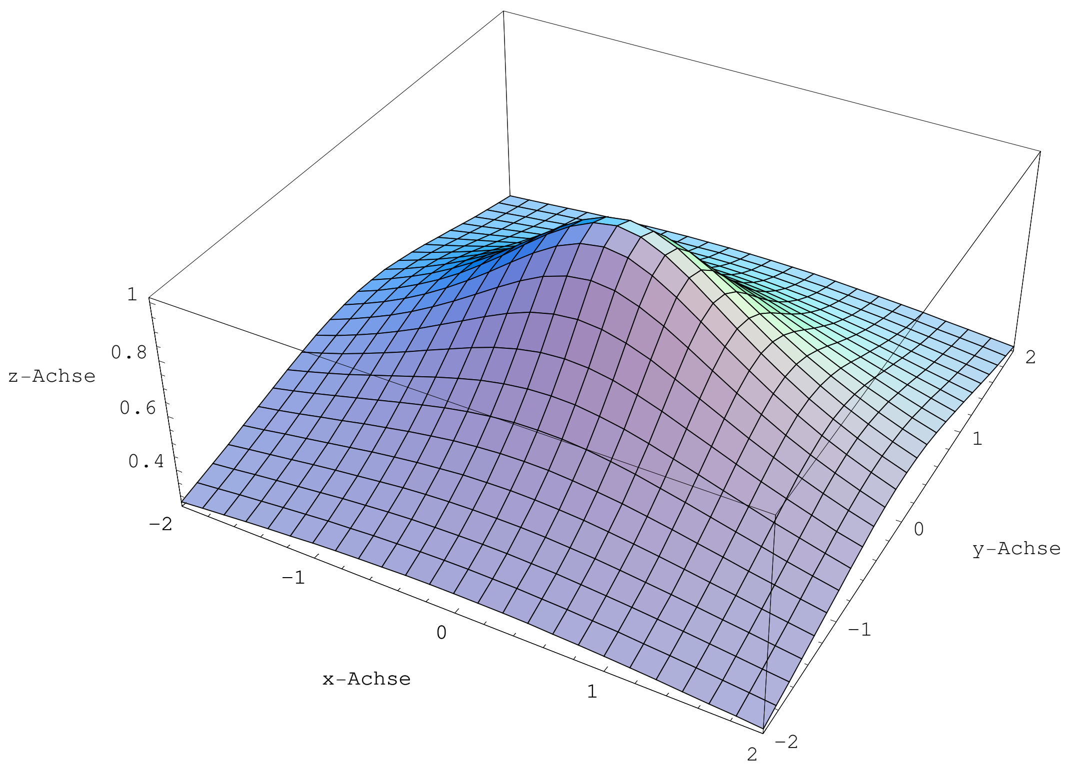
b)

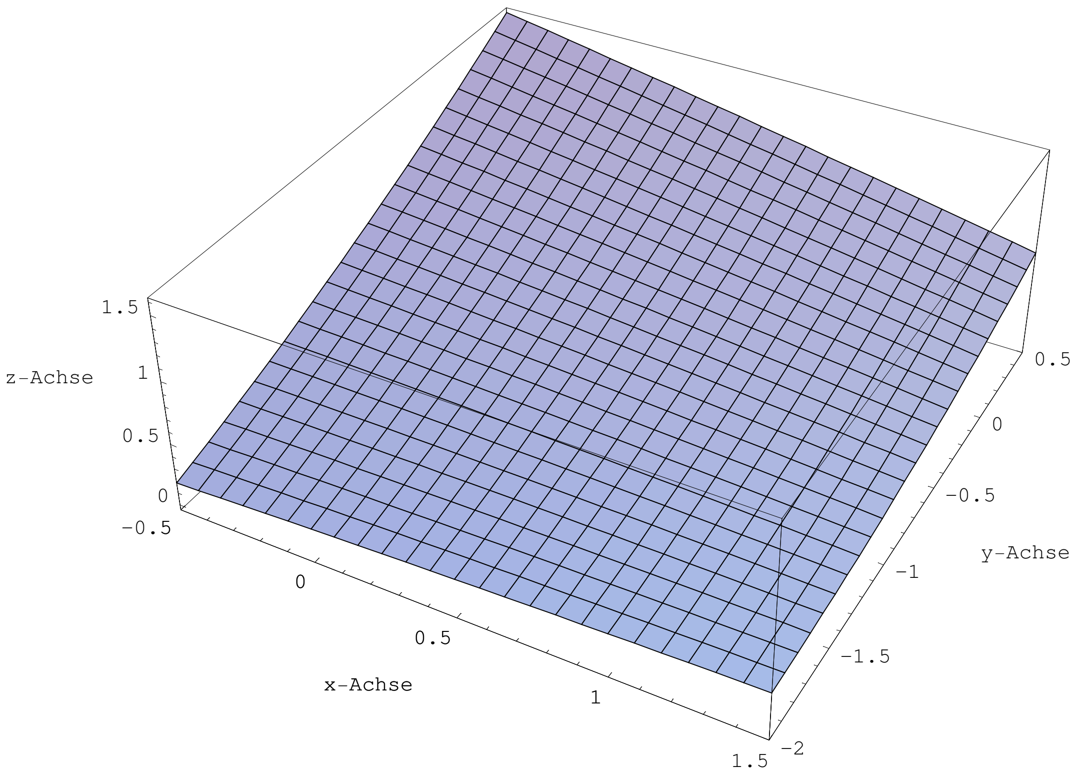
$f(x,y)_{0,0} = 1 - \frac{x^2}{2} - y^2$

Test:

$f(0,0) = 1$

$\tilde{f}(0,0) = 1$

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0
-0.5
0
0.5
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1
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-2
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-1
y-Achse
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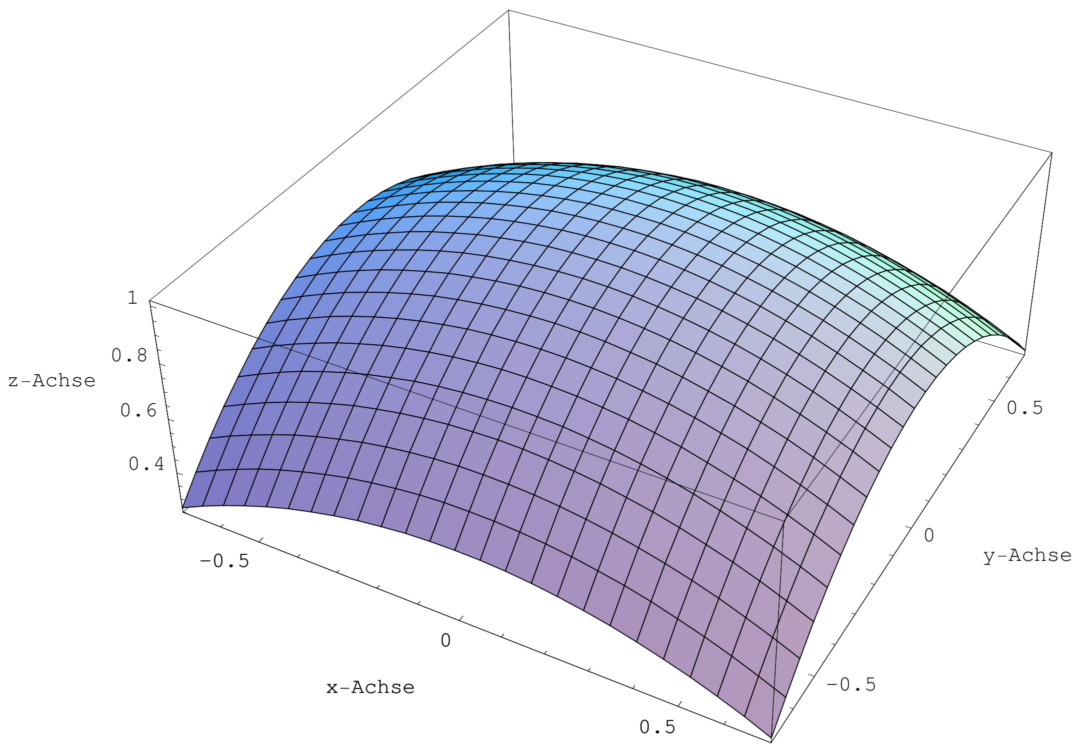
0
0.5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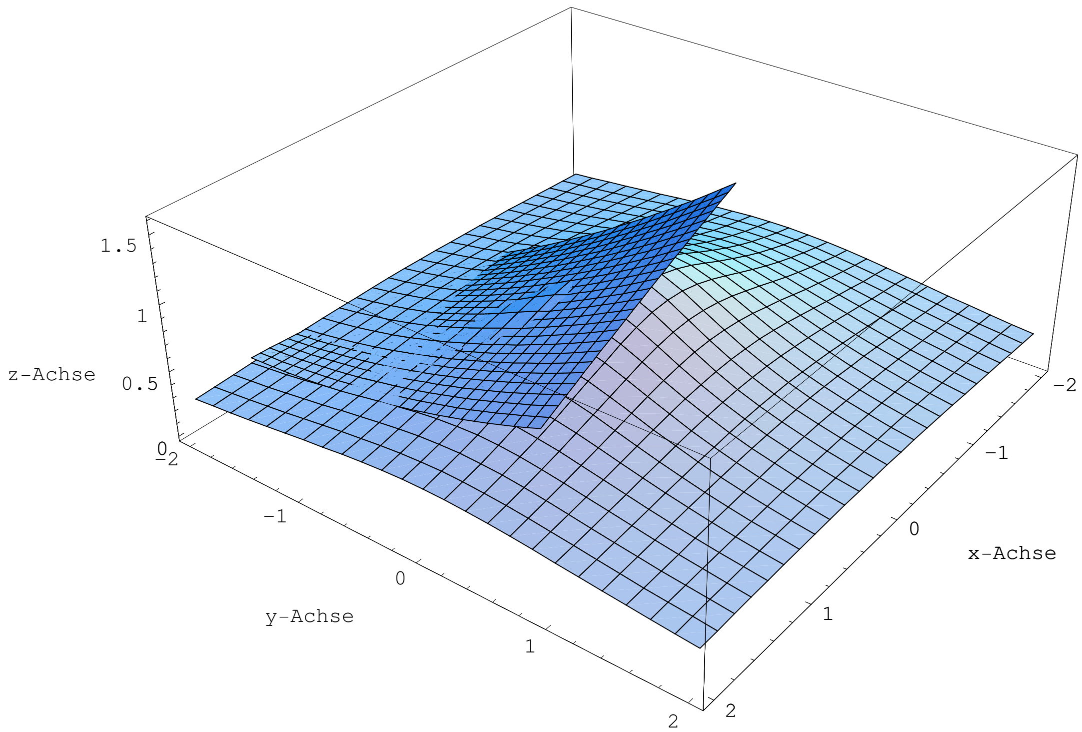
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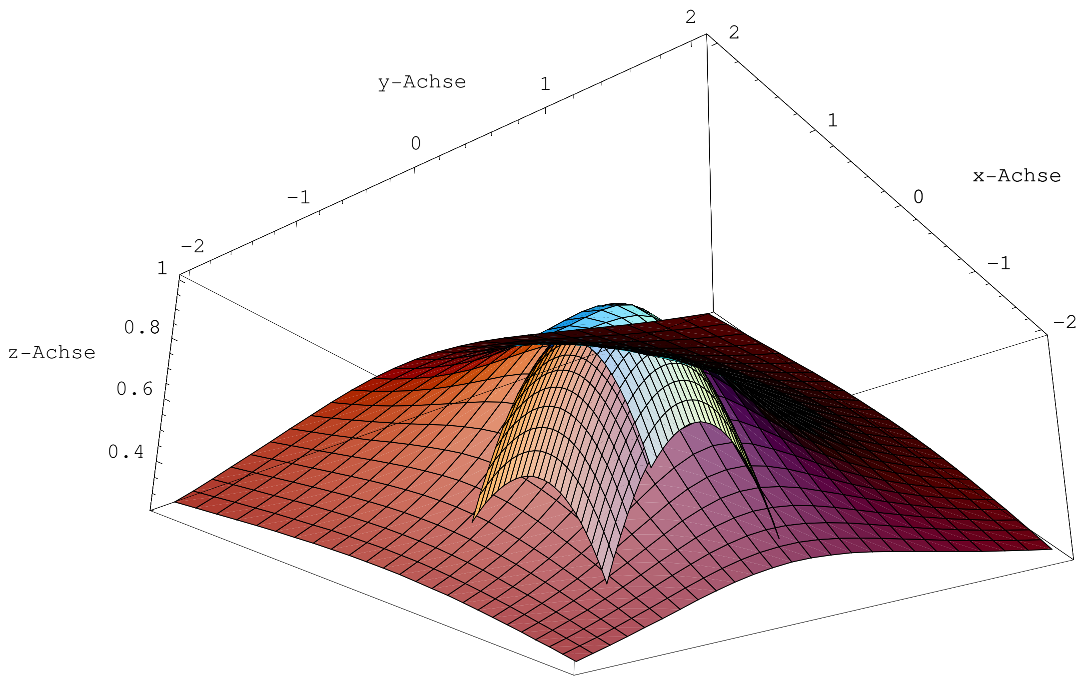
y-Achse
-2
-1
0
1
2
x-Achse


y-Achse
-2
-1
0
1
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x-Achse
2
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0
-1
-2
z-Achse
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0.6
0.4